Sibyllines as matricide] who will persecute the plantation, which the twelve apostles of the beloved have planted and of the twelve one will be given into his hands." I hope Dr. Smith will not deny the great prevalence of the Nero-*redivivus* legend in early Christian circles.

7. Furneaux aptly remarks that the statement of Suetonius concerning the punishment of Christians occurs among a whole list of police regulations for which Nero is commended. This may account for the short wording.

8. As the tilt between Dr. S. and myself may fall into the hands of some who know me personally, I will say that I lay no claim to either a Ph. or D.D., for which my opponent erroneously assumes me.

A. K.

REMARKS ON DR. CARUS'S VIEW CONCERNING GEOMETRY.

In an interesting essay published in *The Monist* of January, 1910, Dr. Carus has attempted to explain the nature of mathematical thought. Putting aside other points, he has mainly endeavored therein to establish "the foundation of geometry without resorting to axioms," which we could not but receive with hearty approval and close attention, because hitherto we have been compelled to proceed with some set or other of axioms, or rather assumptions, as we prefer to call them. If we could ever do away with them, how glad we would be! Nothing else in the domain of mathematics,—nay of any subject in the entire scope of science, could ever afford greater satisfaction to our esthetic requirements by which we are seeking simplicity in our scientific thought. But the case is not simple. We must first enter into a critical examination before we can give assent or dissent to this enticing view of Dr. Carus.

On page 50 of his article we read: "If my conception of mathematics is true we do not need in geometry a certain number of primitive ideas supposed incapable of definition, and a certain number of primitive propositions or axioms, supposed to be incapable of proof."

All this would be very well if it were really true as Dr. Carus maintains. In his Conclusion he feels confident that he has "furnished a conception which satisfies all demands and will be conceivable for all practical purposes," and further that "in the main (his) solution is on the right track." But in spite of all he has said we are compelled to doubt whether he is certainly right. Mathe-

maticians who are interested in the philosophical considerations of their subject would perhaps not be readily persuaded that their theoretical demands have been satisfied by this conception thus prominently set forth by a celebrated philosopher.

If Dr. Carus desires to do away with all axioms, he must base his considerations upon something, or however gifted he be in the art of thinking, he could not build his castle entirely in the air. Thus a cornerstone of his construction lies in his conception of motion. On pp. 37-38 he says:

"We cancel in thought everything particular which comprises all things concrete, be they of matter or energy, and retain only our mental faculty of doing something, including a field of action implied by the possibility of moving about."

Here Dr. Carus has unconsciously introduced an assumption or assumptions. Does he not assume "the possibility of moving about"? The form of his assumption becomes exceedingly clear when he says: "We can move in any direction and everywhere without end" (pp. 39-40). Moreover this statement is not a single assumption only, but it contains a group of assumptions.

Of the numerous assumptions Dr. Carus has tacitly made in the course of his argument, we shall content ourselves to point out a single one. He says on page 40, that "we can draw straight lines in different directions." It is clear that this statement implies an assumption. We shall not speak of various primitive ideas employed by Dr. Carus, that appear to us to be incapable of definition, and stated without any attempt at description.

"Mathematics is a creation of pure thought," Dr. Carus rightly remarks (p. 34). "We do not find a plane anywhere in actual life, we construct it; and in the same sense straight lines and right angles are the products of our construction" (p. 41). All these statements recommend themselves to us as very just, but Dr. Carus does not seem to be always considering geometry in such a purely *a priori* way. In his opinion, "motion is indispensable for any space conception" (p. 72). But what is motion as he conceives it? Does it not seem to be more "concrete" than to be a "pure thought"? It may well answer for the orientation of our conception of a physiological space; it is nevertheless not always necessary for our purely mental construction of mathematical space, as we can see in the different systems actually established by various mathematicians.

He says further (p. 74) that "after all, our notion of space is ultimately based on the self-observation of our own motion; (and)

without motion no space-conception." This may be very true, and we are highly interested with the deep significance of the statement. But it applies only when we have to investigate the origin of our space-conception; it is not positively necessary in our *a priori* construction of any system of geometry. At any rate the idea of motion need not be very conspicuous in such a construction. His statement is of profound significance only with reference to the statement: "Our notion of space is ultimately based on our senses. Without senses no space-conception."

Despite all that, however, Dr. Carus maintains (p. 74), "Pure mathematics does not depend upon the senses but is the product of the mind." If this is so, will it not be possible for us also to form our purely formal conception of space in our mind without resorting to any notion of motion, however conspicuously the latter may have contributed in originating the notion of space in the more or less physiological ground of the formation? This is certainly the reason why motion has not played a conspicuous part in the construction of the now existing systems of geometry.

It is true, that Dr. Carus does not refer to real motion, for on pages 71-72 he says, "This general idea of motion....is not real motion, but the thought of motion." But it is very doubtful whether we are able to conceive lines, angles, triangles etc., as "the purely *a priori* constructions of it."

Notwithstanding all that he has said, I cannot help wondering, if he were not thinking in a more or less "concrete" manner, not in "pure thought" only? His notion is true perhaps "only so far as our physiological space-conception is concerned." In any case Dr. Carus is unknowingly prepossessed of a conception of space in a way analogous to the Euclidean system, which is endowed with something of objective concreteness. We shall hear what he himself says (p. 75):

"We are not able to visualize some of the non-Euclidean spaces, which means we cannot form definite sense-perceptions of them."

Here it appears he is assuming that Euclidean space has been ratified by our senses. Further he says on page 74:

"If rational beings, differing from ourselves, have developed on other planets, they might have different notions of physiological space than we have, but they would have the same logic, the same arithmetic, the same geometry, and all the complications derived therefrom."

It is very strange that Dr. Carus should consider there ought

to be only one geometry, whereas we have various systems. We who inhabit the surface of one and the same planet have already constructed different geometries, and so why should there not be a possibility of the inhabitants of other heavenly bodies constructing other systems than one of those common among us? There may be beings who have attained a much higher degree of evolution than we; their mental faculties may transcend ours in an incredible degree of perfection. Are we not then utterly incapable of even imagining what kind of space-conception they may have formed? Dr. Carus's position is too dogmatic when he uses such a statement as that above quoted.

As to arithmetic, there may be various systems, such as those, for instance, where the laws of association or commutation do not hold.

Dr. Carus says on page 46:

"But if space is a scope of motion, I cannot think of a space that is limited. Spherical space ought to be conceived as possessed of a spherical drift, but for that it ought to be infinite. If it is not infinite, I would ask the question, what is outside?"

Here the Euclidean space is most evidently predominating in the mind of the author, and in consequence he proves to be prejudiced in his considerations. A finite space is only finite; there need be nothing which would involve any conception requiring us to think of what is outside. If we could think of what is outside a finite space, the space would not be finite. Being prepossessed with the conception of the infinite Euclidean space in his mind he is little entitled, it appears, to truly conceive the intrinsic significance of a finite space.

If Dr. Carus says on page 49, "since....there are no points, lines, surfaces, planes, etc., in the objective world, it is obviously impossible to test the truth of Euclidean propositions by actual measurement," this would lead theoretically to the conclusion that any geometrical systems ever conceived in pure thought are all correct in their *a priori* significance. But if we were to consider space as finite and that the length of a whole straight line were not greater than the circumference of the earth's equator, for instance, although this might be logically very correct, it would never answer for practical purposes. If however geometrical systems are constructed to suit the demands of our actual life, we must make a selection as to the best system or systems that would be most convenient for our practical or concrete life. As a matter of course

pure mathematics has little or nothing to do with these things; but in order to secure the concrete application of geometrical systems we must first apply the *a posteriori* judgment of experience. Nothing obliges us to conclude that geometry is inapplicable to concrete purposes, because no such things as points, etc., are found in the actual world.

If the geometrical space be "a universe of pure thought" and yet "a model" serving "for any possible formation, fictitious or real," it would be only too evident that a model could be tested as to whether it would answer our purpose or not.

Dr. Carus condemns the tendency which he calls" experimentalism" met with in some mathematicians, who have raised questions such as these: 'Will not a straight line finally, after billions of miles, ...return into itself?' or, ...'Are the opposite angles in a parallelogram really equal?' or....'Is space Euclidean or non-Euclidean?'...." (pp. 34-35). Dr. Carus takes all these as proving "that those who propose them....do not understand anything of the foundations of mathematics" (p. 35). But here Dr. Carus, it seems, has confounded theoretical considerations with the practical applications of the theories. Some mathematicians, like Poincaré, think that every geometrical system has a significance for us, while others, among whom I may mention L. Harzer, believe otherwise, imagining that actual or objective space may be really limited. Which way of thinking is the better of the two, is a subject which we are not yet able to decide. When I speak in this way, Dr. Carus and his disciples may count me among those who do not understand the foundations of mathematics. I may well be among them; but in my opinion the question lies altogether outside of the domain of pure mathematics and only concerns the practical side of life. A logical construction and its practical application must not be confounded in any case.

For Dr. Carus "both objective existence and our thought...will be analogous" (p. 39), if consistency dominate both. This is certainly the positivist's view and can exercise little authority over those who are not upholders of the positivistic principles. There is consistency between objectivity and our thought, because the former is systematized by the latter. It is therefore not proper to conclude that both are analogous because consistency governs both.

It is very natural that Dr. Carus who is a positivistic philosopher should consider "the formal laws of the universe" as "a part of objective reality." But formal laws have no further significance for

us than as they are developed in our subjectivity. The idea is as absurd as if we should say that the number three is a part of a group of three persons. Three is not in any way comparable with three persons.

Dr. Carus is very right when he says (p. 63):

"The problems concerning the foundations of geometry and of mathematics in general are by no means so definitely settled that one solution may be said to have acquired the concensus of the competent, and for this reason I feel that a little mutual charity is quite commendable."

Thus, if I may differ somewhat in opinion from Dr. Carus, I must openly beg his charity for advocating my own views against him. I may have been led to these discussions "by an enthusiasm as strong as the zeal of religious devotees which....has a humorous aspect," but I am of the firm belief that they will perchance "serve to widen the horizon of his views," although not endowed with the positive power of "reversing, antiquating or abolishing the assured accomplishment" of Dr. Carus.

With us it is *never* "strange that the nature of man's rationality is by no means universally recognized." It seems very natural that "opinions vary greatly concerning its foundation and its origin." We are quite satisfied with the coexistence of various different systems, and so we shall be always happy to receive varying criticisms.

YOSHIO MIKAMI.

OHARA IN KAZUSA, JAPAN, March, 2, 1910.

EDITORIAL COMMENT.

On a first perusal of Mr. Yoshio Mikami's criticism of my views concerning the foundations of geometry, I thought that no reply would be needed for any one who has read my main expositions of the problem, the article in question as well as my books *Kant's Prolegomena* and *The Foundations of Mathematics*. But I am anxious to let every criticism receive consideration, and so I take pleasure in publishing Mr. Mikami's remarks. Since, however, many of our readers have not read the writings under discussion, I will briefly point out why Mr. Mikami's arguments fail to apply to my position.

It is true enough that I propose to lay the foundation of geometry without having recourse to axioms. However I have not for that reason, as Mr. Mikami says, "unconsciously introduced an assumption or assumptions," but I build all the formal sciences upon the facts of our own existence. In doing so I simply follow the